On the capacity of MIMO Rician Channels

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Presentation outline

1. Part 1 - MIMO Rician channels

2. Part 2 - Research project
Part 1 - MIMO Rician channels
- Introduction
- Capacity of MIMO Rician channels

Part 2 - Research project
Introduction

- MIMO wireless communication systems have great potential of providing excellent spectral efficiency.
- Theoretical limits of such systems have been well studied for the i.i.d. Rayleigh scenario, but in practice there is often a line-of-sight path between the transmitter and the receiver.
- In this case, the channel model is best described by a Rician channel model, where the elements of the channel matrix are complex gaussian random variables with non-zero means.
- It is important to evaluate the capacity of a system in a Rician fading environment, in order to obtain a more realistic assessment of the potential of MIMO channels.
Consider a single-user Gaussian channel, with $N_t$ antennas at the transmitter and $N_r$ antennas at the receiver. The channel can be modeled as

$$y = Hx + w,$$

where $y$ is the received word and $H$ is the channel matrix, such that $H = \overline{H} + H_d$. $x$ is the transmitted word and $w$ the AWGN noise.

**Definition of the capacity**

The channel capacity is defined as the maximum rate at which data can be transmitted at an arbitrarily small error probability, given by:

$$\max_{p(x)} I(x, y),$$

where $p(x)$ is the input distribution and $I(x, y)$ is the mutual information between the transmitted and received vectors.
The mutual information is given by:

\[ I(x, y) = \log_2(\det(I_{N_r} + \frac{1}{N_0}HK_xH)) \]

where \( K_x \) is the input’s covariance matrix.

In the case of Rayleigh fading (or no channel state information (CSI) at the transmitter with Rician fading), it has been conjectured that the optimal covariance matrix is

\[ K_x = \frac{P_T}{N_t}I_{N_t}, \]

where \( P_T \) is the total available power.
The mutual information, as presented above, is a random variable depending on channel realizations; its statistics may be interesting to study (mean, variance, complementary cumulative distribution function).

It has been analytically proved that the mutual information of an i.i.d. Rayleigh MIMO channel is approximately gaussian, even when the number of antennas of the system is small.

Furthermore, it has been observed that this approximation is also well suited for MIMO Rician channels.

When the channel is ergodic, only the first-order statistics os the mutual information are needed to characterize the channel. We have:

$$C = E[J].$$
When the channel is non-ergodic channels, the capacity in the usual sense is not applicable.

For this matter, a way of studying the channel’s limits is the outage probability, defined as:

\[ p_{\text{out}} = \min_{K_X | \text{Tr}[K_X] \leq P_T} \mathbb{P}\{\log_2(\det(I_{N_T} + \frac{1}{N_0}HK_XH)) < R\}. \]

Using the gaussian approximation for the mutual information the cumulative distribution function can serve as an upper bound for the outage probability.

This requires the first two-moments of the mutual information approximation.
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   - Introduction
   - Motivation
   - Preliminary approach
   - Present and future work
Introduction

- In wireless systems, two design techniques exist, that are concurrent and also, normally opposed to each other:

1. Resource allocation - Full CSI at the transmitter
   - Treats problems such as optimization criteria, mainly mutual information or signal-to-noise ratio.
   - A well known technique is *waterfilling*.

2. Diversity techniques - No CSI at the transmitter
   - The sender does not know anything about the channel;
   - Certain techniques are applied, in order to maintain some reliability in the sent information; we cite: space-time codes attaining the Diversity-Multiplexing tradeoff.
Motivation

In our context, the sender has partial channel state information, i.e., it knows the average part of the channel, but not the diffuse part.

The two above techniques must thus co-exist.

We propose a joint design of the power allocation in the system and the coding, for diversity.

3. Resource Allocation + Diversity techniques - Partial CSIT

i. The model of the channel must be carefully studied.

ii. How can optimization and space-time coding be used together.
Preliminary approach

• In a first approach, we suppose the transmitter knows the direct path of the Rician channel.

• In order to minimize the outage probability, an optimization must be applied, using this direct path: *waterfilling*.

• This optimization, however, must maintain the diversity order, introduced by the space-time code.

• Another important issue to consider is **link adaptation**: scheduling techniques must be introduced.

• The capacity of each sub-channel must be separately considered and data rates should be individually adjusted.
Some first results

Considering a 2 x 2 MIMO system, with a Rician channel matrix, preliminary results show:

Outage probability, $K=10$ dB

![Graph showing outage probability vs. SNR]
Present and future work

- Implementation of a decoder, using a STBC in order to evaluate effective word error rate of the system, under a MIMO Rician channel.

- Optimization of the covariance matrix.

- Study of diversity constraints on the system, given different scenarios.

- Generalization towards cooperative channels (relay and interference channels.)